Efficient Compressive Sensing for Blind Source Separation of Audio Signal

Abstract: This work presents a novel framework for separating and reconstructing audio signals from compressively sensed linear mixtures. In most cases of audio signal processing only mixture of audio sources are available for observation. So separation of sources is essential. Conventional approach for data acquisition use Nyquist Sampling Theorem. We propose an approach, which use compressively sensed measurements of the mixtures for separation ie; the Blind Source Separation is applied to signal mixtures in lower domain. Moreover our approach has lower computational complexity. The main contribution of this paper lies in proposing an efficient approach for the compressive sampling of data through designing the sensing matrix in Structurally Random Matrix(SRM) form, which possess lower memory and high speed and the separation of the signals via Blind Source separation(BSS) without reconstruction of the mixtures. The separation is done considering source as independent and Independent Component Analysis is employed which utilised Gradient Ascent algorithm here.

Keywords: Compressive Sensing, Structurally Random Matrix, Blind source Separation, Independent Component Analysis, Gradient Ascent Algorithm.

I. INTRODUCTION

Compressive sensing (CS) is a new approach to data acquisition first proposed by Candes, Romberg and Tao and Donoho. Traditional signal acquisition is based on the Nyquist sampling theorem, which shows that the sampling rate must be no less than twice the maximum frequency component of the signal. Here as signal frequency increases, the sampling rate becomes faster and faster, which presents a huge challenge to sampling devices as well as in data storage and transfer. For example: when we have limitations on the number of data capturing devices, measurements are very expensive or slow to capture such as in radiology and imaging techniques via neutron scattering or in some other cases like the audio source acquisition on a spaceship or space station. In such situations, CS provides a promising solution. Compressive sensing has witnessed an increased interest recently courtesy high demand for fast, efficient and in-expensive signal processing algorithms, applications and devices. In many field, such as speech recognition, network anomaly detection, and medical signal processing, only the mixtures of source signals are available for compressive sampling.

So any operation i like to perform on the signal, first require it to be separated from the mixture. As already said audio signals in real time are available as mixtures of two or more sources. The mixing process of source signals in BSS problems can be sorted into several models, such as the convolved mixture model, the instantaneous linear mixture model, or the nonlinear mixture model. In this paper, we consider only the instantaneous linear mixture model.

The recovery of the mixing matrix and source signals from the compressive measurements of mixing signals is one of the main tasks. Recovering mixing parameters and source signals only from the mixed observations without having prior information on both source signals and the mixing process is often referred to as blind source separation (BSS).
Blind mixing matrix estimation is one of the most important steps in BSS, which significantly affects the recovery accuracy of the source signals. Independent component analysis (ICA) is one of the most popular methods for solving BSS problems. In this paper, we consider the blind mixing matrix recovery problem in the compressive measurement domain. There is an underlying algorithm, which can separated into two procedures comprising of compressive sensing and blind mixing matrix recovery, i.e. Step 1 is to reconstruct the mixture signals from the observed compressive measurements, while step 2 is recovering the mixing matrix from the mixture signals. Note that this method needs a full reconstruction of mixture signals. However, in many signal processing problems, it is possible to directly extract attributes of the signal from the compressive measurements and a full reconstruction is not necessary.

This paper proposes a novel recovery algorithm which estimates the mixing matrix in the compressive measurement domain without ever reconstructing the mixture signals. This paper transfers the gradient ascent algorithm for source separation to compressive measurement domain which is developed to perform efficiently and faster using Structurally Random Matrix (SRM).

The rest of this paper is organized as follows.

Section II depicts the theoretical background of this work. While detailed description of the proposed algorithm is provided in section III. Section IV presents simulation results. Finally the conclusion is given in section V.

In chapter 2, a detailed literature survey will be provided. The details of different techniques used for the sensing matrix design and various blind source separation techniques are provided. In chapter 3, the problem is formulated. In chapter 4, mathematical morphology is briefly described and solution to the problem is given. In Chapter 5, gives as the reference

II. PROPOSED ALGORITHM

In the proposed algorithm the reconstruction of the mixtures in compressive domain is eliminated through performing the blind separation of the mixtures in lower domain itself. This will help in obtaining reduction in the no of computation and increase in speed. Since audio signals are assumed to be independent, Independent Component Analysis which is one of the most popular methods is considered for separation. Various algorithms are available in ICA and this work utilized Gradient ascent algorithm for better performance.

Here audio signal is assumed to be sparse in Discrete Cosine Transform and mixtures are developed in non-noisy linear mixture model. The algorithm aims in developing a sensing matrix which efficiently acquire sparse elements from signal samples. To the signal in sparse domain blind source separation is applied to separate each sources. Each mixture is created from source vectors through a mixing matrix. So to estimate the sources from mixtures it is better to estimate the mixing matrix or its inverse, say unmixing matrix. Here in gradient algorithm the unmixing matrix W is estimated as one which seeks to maximize the entropy H(Y ) of the signals Y = g(xW). The function g() is a nonlinear function. Given an initialization of W, the gradient ascent method iteratively adjusts W to maximize the entropy of Y = g(xW). H(Y ) is described as follows

\[ H(Y) = H(x) + E[\sum_{i=1}^{M} \ln g'(Wx)] + |\ln |W|
\]

We follow all the notation in the compressive sensing literature and represent the signals as column vectors. Here the audio mixture is assumed to follow instantaneous linear BSS model which is given by

\[ X = SA \]

where S = [s1, s2, s3, ..., sn] ∈ R^{mxn} denotes the data matrix of m sources with n samples (m << n) , A = [a1; a2; a3; ...ak] ∈ R^{mXm} is the mixing matrix of full rank and Y = [x1; x2; x3; ...xn] ∈ R^{nXk} represents the K linear mixtures of S. The task of BSS method is to estimate the source S, when we are provided only the mixtures Y. Efficient way to solve this problem is to create an unmixing matrix W, with which an estimate of s and A can be made as
\[ s' = xW \]
\[ A = W^{-1} \]

Compressive Sensing (CS) theory indicates that, if any audio signal or its mixture is sparse or compressible in some basis, it can be recovered with a high accuracy from a few numbers of its measurements. Consider \( X = [x_1; x_2; \ldots; x_m]^T \) be a \( K \) sparse signal with length \( N (K << N) \). The sparse basis \( \Psi \) be an \( N \times N \) matrix, with the sparse coefficient vector \( \eta \in \mathbb{R}^N \). The signal can be denoted as follows

\[ x = \varphi \eta \]

Let \( || \eta ||_o \) be the \( M \times N \) measurement matrix, where \( M < N \). The observation vector \( y \) consist of \( M \) consist of \( M \) linear projections of \( x \).

\[ y = \varphi x = \varphi \Psi \eta = \eta \]

where \( \varphi \Psi \eta \) is called sensing matrix. Each audio mixtures \( x_i(t) \) can be written as follows:

\[ x_i(t) = \varphi \eta_i \quad (i = 1,2,3,\ldots,m) \]

Thus all the mixtures are \( K \) sparse and are constructed from the same \( K \) elements of \( \Psi \), with different coefficients. So we take benefit of the same measurement matrix \( \varphi \) to take compressive measurement of all the mixtures as they all have same sparse level. Hence the compressive observation set can be denoted as follows:

\[ y = [y_1, y_2, y_3, \ldots, y_m] \]

### III. Problem Description

The work finds its obstacles in designing the sensing matrix for compressive measurements and in designing the unmixing matrix for blind source separation. Sensing matrices come mainly as a random projection or a random matrix of i.i.d random variables generated from a Gaussian or Bernoulli distribution. This family of sensing matrix is well-known as it is universally incoherent with all other sparsifying basis. For example, if \( \varphi \) is a random matrix of Gaussian i.i.d entries and \( \Psi \) is an arbitrary orthonormal sparsifying basis, the sensing matrix in the transform domain \( \varphi \Psi \) is also Gaussian i.i.d matrix. This universal property of a sensing matrix enables us to sense a signal directly in its original domain without significant loss of sensing efficiency and without any other prior knowledge. Also random projection approaches the optimal sensing performance of \( M = O(K \log N) \).

However, it is quite costly to realize random matrices in practical sensing applications as they are complex in computation and have huge memory buffering due to their unstructured nature. For example, to process a 512 x 512 image with 64K measurements, a Bernoulli random matrix requires nearly giga bytes storage and giga-flop operations. Then the size in the case of audio is infinitely high causing both the sampling and recovery processes very expensive and in many cases, unrealistic.

Remaining challenges for CS in practice is to design a CS framework that has the following features: Optimal or near optimal sensing performance: the number of measurements for exact recovery approaches the minimal bound, i.e. on the order of \( O(K \log N) \); Universality: sensing performance is equally good with almost all sparsifying bases; Low complexity, fast computation and block-based processing support: these features of the sensing matrix are desired for large-scale, real time sensing applications; Hardware/Optics implementation friendliness: entries of the sensing matrix only take values in the set \( \{0,1,1\} \).
IV. Solution Procedure

This paper provides a novel approach for the efficient design of the sensing matrix that satisfies the above wish list. It is called Structurally Random Matrix (SRM). It is defined as the

$$\sqrt{\frac{N}{M}} DFR$$

where:

1. $R \in \mathbb{R}^{N \times N}$ is either a uniform random permutation matrix or a diagonal random matrix whose diagonal entries $R_{ii}$ are i.i.d. Bernoulli random variables with identical distribution $P(R_{ii} = (0,1,1)) = \frac{1}{2}$

2. $F \in \mathbb{R}^{N \times N}$ is an orthonormal matrix that, in practice, is selected to be fast computable such as popular fast transforms: FFT, DCT, WHT or their block diagonal versions. The purpose of the matrix $F$ is to spread information (or energy) of the signals samples over all measurements

3. $D \in \mathbb{R}^{M \times N}$ is a subsampling matrix/operator. The operator $D$ selects a random subset of rows of the matrix $FR$. If the probability of selecting a row $P$ (a row is selected) is $M/N$, the number of rows selected would be $M$ on average. In matrix representation, $D$ is simply a random subset of $M$ rows of the identity matrix of size $N \times N$.

The scale coefficient $\sqrt{\frac{N}{M}}$ is to normalize the transform so that energy of the measurement vector is almost similar to that of the input signal vector. With the SRM method, the computational complexity and memory space required is independent with the number of measurements $M$. The only requirement is to store the diagonals of $D$ and $R$ matrix other than storing $D,F,R$ matrices. This reduces the hardware required for designing this matrix and hence less computations. Depending on specific applications, SRM can offer computational benefits either at the sensing process or at the signal reconstruction process.

The second phase of the work lies in separation of the observed mixtures through developing a mixing matrix followed by estimation of source signals using this. Since the mixtures are now in compressed form Independent component Analysis can now be applied to mixtures in lower domain. Statistical properties of higher order of mixtures are applied to retrieve the mixing matrix when no prior information regarding the mixture or matrix are provided. Since we are using independent component analysis for blind separation of audio, it requires the source audio signals which contribute to this mixture to satisfy two main properties.

1) All the audio sources must be independent of each other

2) Gaussian sources are not encouraged, but at most one source can be allowed.

Usually signals are considered are independent if information on one audio signal sample does not give any information about other audio samples. CA method deals with separation of signals through utilizing its higher order statistics like entropy. Drawback of audio signal is that it lack higher order moments and we need to satisfy with its mean and variance only. So if more no of sources is Gaussian the mixture also tend to be Gaussian which make it impossible to separate it through ICA. We usually assume audio signal are statistically independent and also sparse in Discrete Cosine domain. Checking the kurtosis value is the best way to estimate the gaussianity of signal

Different algorithm is available in ICA method. The FastICA algorithm, the Infomax algorithm, and the gradient ascent algorithm are among them. Here gradient ascent algorithm is preferred.
**V. ALGORITHM DESCRIPTION**

In ICA model, the observed mixture signals $x(t)$ can be expressed using the independent source signal as

$$x(t) = As(t)$$

where $A$ is an unknown mixing matrix, and $s(t)$ represents the latent source signals which is supposed to be statistically equally independent. Broad ICA model in the noising case is given as

$$x(t) = As(t) + n(t)$$

The independent components $s(t)$ cannot be directly observed and the mixing coefficients $A$ and the noise $n(t)$ is also assumed to be unknown. Noise is considered negligible. The ICA solution is obtained in an unsupervised way that finds a de-mixing matrix $C$. The de-mixing matrix $W$ is used to transform the observed mixture signals $x(t)$ to give the independent signals. That is:

$$s'(t) = Cx(t)$$

The signals $s'(t)$ are the close estimation of the source signals $s(t)$. If $C = A^{-1}$, then the recovered signals $s(t)$ are exactly the original sources $(s)$. The components of $s(t)$, called independent components, are required to be as mutually independent as possible.

Gradient ascent algorithm can extract source signals from the mixture signals if the source signals are independent or uncorrelated with each other. In the problem of ICA, given a set of mixture signals $x$, we seek an unmixing matrix $W$ which maximizes the entropy $H(Y)$ of the signals $Y = g(Wx)$. We can find an estimate $W$ using gradient ascent method to iteratively adjust $W$ in order to maximize the entropy of $Y = g(Wx)$. $H(Y)$ is described as follow:

$$H(Y) = H(x) + E\left[\sum_{i=1}^{M} (\ln g'(Wx))\right] + \ln |W|$$

The task of this algorithm is to seek an unmixing matrix $W$ from the obtained measurements $y$. Different from the gradient ascent algorithm, let the compressive measurements $y$ replace the mixture signals $x$. Then, iteratively adjust $W$ to maximize the entropy $H(Y)$ of $Y = g(Wy)$.

The proposed algorithm follows as:

Some Pre-processing works are essential to be performed on the available data for simpler ICA estimation. Those techniques are:

- Centering: It is a technique used to center the signal $x$, by subtracting the mean vector $m \ E\{x\}$ so that to make the signal a zero mean variable.

- Whitening: An additional pre-processing technique used to first whiten the observed data. Here before the application of ICA algorithm and after centering, a linear transformation is given to the signal to make a new signal which is white. By white it resembles that the variance is unity as its components are uncorrelated.

The flowchart of this proposed algorithm through Gradient Ascent algorithm is shown in Figure below.
The input mixture signal $x$ is $N$-dimensional in the gradient ascent algorithm, while the compressive measurements $y$ is $M$-dimensional in this proposed algorithm and $M \ll N$. It can be clearly seen that this algorithm will reduce the computation of mixing matrix recovery. Here $Y$ is nonlinear function of $y$ given as the negentropy function. It measures the difference in entropy between a given distribution and the Gaussian distribution with the same mean and variance.

The above shown algorithm can be described as follows in simple steps:

1. Center the data to make its mean zero;
2. Whiten the data to get $x'(t)$;
3. Make $i = 1$;
4. Choose an initial orthogonal matrix for $W$ and make $k = 1$;
5. Make $w_i(k) = [(w_i(k + 1)^T)3] - 3w_i(k-1)$
6. Make $w_i(k) = w_i(k)/w_i(k)$
7. If not converged, make $k = k + 1$ and go back to step (5)
8. Make $i = i + 1$.
9. When $i$ < number of original signals, go back to step (4).

VI. RESULT AND DISCUSSION

Since dealing with audio signals which possess a large no of samples, all processing are done on it after a segmentation. A frame of prescribed length is developed for it. Broad ICA model is considered to contain interference of noise during data acquisition. For the ease of performance it is purposefully eliminated. Sensing matrix is designed through SRM method as for audio signal dimension is quite large. With the SRM method, the computational complexity and memory space required is independent with the number of measurements $M$. The only requirement is to store the diagonals of $D$ and $R$ matrix other than storing $D$, $F$, and $R$ matrices. Discrete Cosine Transform is considered for transformation due to its high compaction.

The Gaussianity of each mixtures are checked using kurtosis as ICA method is only applicable to non-Gaussian signals. Preprocessing methods like centering and whitening are done on signal which makes the signal covariance unity or we say signal components are uncorrelated which helps us to develop an initial unmixing matrix $W$ with most values one. Differential
entropy is generated for the signal with respect to the entropy of Gaussian signal generated with the signal mean and standard deviation. The W matrix is iteratively adjusted using gradient ascent method till it maximizes the entropy of $Y = g(xW)$ where $g(\cdot)$ is a non-linear function.

VII. SIMULATION RESULT

The simulation is carried out using MATLAB 2014b. The code was written to generate mixtures from independent sources as shown on figure 2 and figure 3.

![Figure 2. Independent audio signals](image1.png)

![Figure 3. Mixture of two signals](image2.png)

The mixtures are then transformed to compressive form through compressive sensing framework. The algorithm developed for gradient Ascent will generate the unmixing matrix iteratively. This matrix is utilised to get independent sources as shown in figure 4.

![Figure 4. Separated Signals using ICA](image3.png)
VIII. CONCLUSION

Compressive Sensing has a great capability to acquire signal in compressive form, which other than the usual sampling theorem store only sparse components of audio. The design of sensing matrix through SRM has provided a great reduction in memory and thus high efficiency. Independent Component Analysis here uses the gradient method to separate signals from the mixtures (here in compressed form) to independent source signals. The algorithm applied here is Gradient Ascent Method which utilises the negentropy of signal. The algorithm was applied in this research to separate two independent sources from three mixtures recorded and saved in .wav format.

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